Kinetic Theory

1. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}\text{ (helium)}}{V_{rms}\text{ (argon)}}\right]$ is

2. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat (in kJ) transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take R = 8.3 J/K mole]

3. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done (in joule) by gas is (Gas constant R = 8.31 J/mol − K)

4. An ideal gas occupies a volume of 2 m^3 at a pressure of 3×10^6 Pa. The energy (in joule) of the gas is

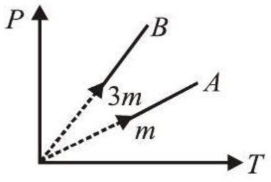
5. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time (in second) between two successive collisions will be

6. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure (in N/m^2) exerted by the gas molecules will be

7. The temperature (in kelvin), at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is [Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K
   Avogadro Number $N_A = 6.02 \times \frac{10^{26}}{kg}$; Radius of Earth: 6.4×10^6 m
   Gravitational acceleration on Earth = 10 ms^{-2}]

8. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed (in m/s) of the molecules will be:

9. Two different masses m and $3m$ of an ideal gas are heated separately in a vessel of constant volume, the pressure P and absolute temperature T, graphs for these two cases are shown in the figure as A and B. The ratio of slopes of curves B to A is



10. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume (in m^3) does it grow, when it reaches the surface which is at a temperature of 35°C ?

11. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat (in J/molK) of mixture at constant volume? (R = 8.3 J/molK)

12. A narrow uniform glass tube 80 cm long and open at both ends is half immersed in mercury. Then, the top of the tube is closed and it is taken out of mercury. A column of mercury 22 cm long then remains in the tube. What is the atmospheric pressure (in cm of mercury) ?

13. A vessel of volume $V = 5.0$ litre contains 1.4 g of nitrogen at temperature, $T = 1800$ K. Find the pressure (in N/m^2) of the gas if 30% of its molecules are dissociated into atoms at this temperature.

14. A gaseous mixture enclosed in a vessel consists of one gm mole of a gas A with $(\gamma = 5/3)$ and another B with $(\gamma = 7/5)$ at a temperature T. The gases A and B do not react with each other and assumed to be ideal. Find the number of gram moles of the gas if γ of the gaseous mixture is 19/13.

15. An oxygen cylinder of volume 30 litre has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drop to 17°C. Estimate the mass (in kg) of oxygen taken out of the cylinder, $R = 8.3$ J $\text{mol}^{-1}K^{-1}$, molecule weight of oxygen $= 32$.

SOLUTIONS

1. **(3.16)** Using $\frac{V_{1rms}}{V_{2rms}} = \sqrt{\frac{M_2}{M_1}}$

$$\frac{V_{rms}(\text{He})}{V_{rms}(\text{Ar})} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$$

2. **(10)** Heat transferred,

$Q = nC_v \Delta T$ as gas in closed vessel

To double the rms speed, temperature should be 4 times

i.e., $T' = 4T$ as $v_{rms} = \sqrt{3RT/M}$

$$\therefore Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

$$\left[\therefore \frac{C_P}{C_V} = \gamma_{\text{diatomic}} = \frac{7}{5} \& C_p - C_v = R\right]$$

or, $Q = 10000\,J = 10\,kJ$

3. **(291)** Work done,

$$W = P\Delta V = nR\Delta T = \frac{1}{2} \times 8.31 \times 70 \simeq 291J$$

4. **(9×10^6)** Energy of the gas, E

$$= \frac{f}{2} nRT = \frac{f}{2} PV$$

$$= \frac{f}{2}(3 \times 10^6)(2) = f \times 3 \times 10^6$$

Considering gas is monoatomic i.e., $f = 3$

Energy, $E = 9 \times 10^6$ J

5. **(4×10^{-8})** Using, $\tau = \frac{1}{2n\pi d^2 V_{avg}}$

$$\therefore t \propto \frac{\sqrt{T}}{P}\left[\therefore n = \frac{\text{no.of molecules}}{\text{Volume}}\right]$$

or, $\frac{t_1}{6 \times 10^{-8}} = \frac{\sqrt{500}}{2P} \times \frac{P}{\sqrt{300}} \approx 4 \times 10^{-8}$

6. **(2)** Rate of change of momentum during collision

$$= \frac{mv - (-mv)}{\Delta t} = \frac{2mv}{\Delta t} N$$

so pressure

$$P = \frac{N \times (2mv)}{\Delta t \times A} = \frac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1 \times 1} = 2\,N/m^2$$

7. **(10^4)** $v_{rms} = v_e$

$$\sqrt{\frac{3RT}{M}} = 11.2\times10^3$$

or $$\sqrt{\frac{3kT}{m}} = 11.2\times10^3$$

or $$\sqrt{\frac{3\times1.38\times10^{-23}T}{2\times10^{-3}}} = 11.2\times10^3 \qquad \therefore \quad v = 10^4 \text{ K}$$

8. **($100\sqrt{5}$)** $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{(273+127)}{(273+227)}}$$

$$= \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore v_2 = \frac{\sqrt{5}}{2}v_1 = \frac{\sqrt{5}}{2}\times200 = 100\sqrt{5}\,\text{m/s}.$$

9. **(3)** $PV = \frac{m}{M}RT$; $\quad\therefore\ \frac{P}{T} = Cm$

or $$\frac{\text{slope of } B}{\text{slope of } A} = \frac{m_B}{m_A} = \frac{3m}{m} = 3$$

10. **(5.275×10^{-6})** At the bottom of the lake, $V_1 = 1\text{ cm}^3$,

$$P_1 = P_a + 40\,m$$
$$= 10.3 + 40$$
$$= 50.3\,m \text{ of water.}$$
$$T_1 = 273 + 12$$
$$= 285K.$$

At the surface, $$P_2 = P_a$$
$$= 10.3\,m \text{ of water.}$$
$$T_2 = 273 + 35$$
$$= 308\,K.$$

Using $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

or $$V_2 = \frac{P_1V_1T_2}{P_2T_1}$$

$$= \frac{50.3\times1\times10^{-6}\times308}{10.3\times285}$$
$$= 5.275\times10^{-6}\text{ m}^3$$

11. **(17.4)** $[C_v]_{\min} = \frac{n_1[C_{v_1}]+n_2[C_{v_2}]}{n_1+n_2}$

$$= \left[\frac{2\times\frac{3R}{2}+3\times\frac{5R}{2}}{2+3}\right]$$

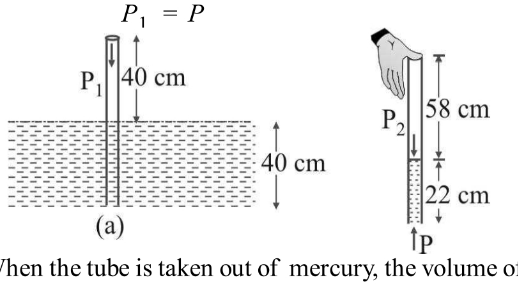
$= 2.1$ R

$= 2.1\times8.3$

$= 17.4$ J/mol–K

12. **(70.9)** Suppose A is the area of cross-section of the tube and P is the atmospheric pressure. When half tube is inside mercury then volume of air in the tube.

$$V_1 = A \times 40 = 40\,\text{A cm}^3$$
$$P_1 = P$$



When the tube is taken out of mercury, the volume of air,

$$V_2 = A \times 58 = 58A$$

If P_2 is the pressure of air in the tube, then

$$P_2 + 22 = P$$

or $$P_2 = P - 22$$

Now by Boyle's law, we have

$$P_1 V_1 = P_2 V_2$$
$$P \times 40\,A = (P - 22) \times 58\,A$$

or $$18\,P = 22 \times 58$$

$\therefore$ $$P = 70.9\ \text{cm.}$$

13. **(1.94×10^5)** Mass of molecular nitrogen

$$= \frac{70}{100} \times 1.4 = 0.98\ \text{g}$$

Mass of atomic nitrogen

$$= \frac{30}{100} \times 1.4 = 0.42\ \text{g}$$

Number of moles of molecular nitrogen,

$$n_1 = \frac{0.98}{28} = 0.035$$

Number of moles of atomic nitrogen

$$n_2 = \frac{0.42}{14} = 0.03$$

The pressure of the gas = pressure exerted by molecular nitrogen + pressure exerted by atomic nitrogen

i.e., $$P = P_1 + P_2$$

$$= \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = \frac{(n_1 + n_2)RT}{V}$$

$$= \frac{(0.035 + 0.03) \times 8.31 \times 1800}{5 \times 10^{-3}}$$

$$= 1.94 \times 10^5\ \text{N/m}^2.$$

14. **(2)** For two gases, we can write

$$\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_{mean} - 1}$$

Here, $$n_1 = 1 \text{ and } n_2 = ?$$
$$\gamma_1 = 5/3 \text{ and } \gamma_2 = 7/5$$

Substituting these values in above equation, we get

$$\frac{1}{\frac{5}{3}-1}+\frac{n_2}{\frac{7}{5}-1} = \frac{1+n_2}{\frac{19}{13}-1}$$

or $$\frac{3}{2}+\frac{5n_2}{2} = \frac{13(1+n_2)}{6}$$

or $$3(3+5n_2) = 13(1+n_2)$$

$$9+15n_2 = 13+13n_2$$

$$n_2 = 2 \text{ gram mole.}$$

15. (0.141) Given,

$$P_1 = 15+1$$
$$= 16 \text{ atm (absolute)}$$
$$V_1 = 30\times 10^{-3}\ \text{m}^3,$$
$$T_1 = 273+27$$
$$= 300\,K.$$
$$P_2 = 12 \text{ atm},$$
$$T_2 = 273+17$$
$$= 290\,K.$$

We have $$PV = \frac{m}{M}RT$$

$\therefore$ $$m = \frac{P_1 VM}{RT_1}$$

$$= \left[\frac{(16\times1.013\times10^5)\times(30\times10^{-3})\times(32\times10^{-3})}{8.31\times300}\right]$$

$$= 0.624 \text{ kg.}$$

Finally $$m' = \frac{P_2 VM}{RT_2}$$

$$= \left[\frac{(12\times1.013\times10^5)\times(30\times10^{-3})\times(32\times10^{-3})}{8.31\times290}\right]$$

$$= 0.484 \text{ kg}$$

The mass of the oxygen taken out = $m - m'$

$$= 0.624-0.484$$
$$= 0.141 \text{kg.}$$